Hybrid Doublet Lattice/Doublet Point Method for Lifting Surfaces in Subsonic Flow

Walter Eversman*
University of Missouri—Rolla, Rolla, Missouri 65401
and
Dale M. Pitt†
McDonnell Aircraft Company, McDonnell Douglas Corporation, St. Louis, Missouri 63166

A hybrid unsteady lifting surface theory combining the best features of the traditional doublet lattice method and the more recent doublet point method has been developed. The hybrid code will handle unequal strip widths and nonparallel intersecting lifting surfaces and retains most of the computational efficiency of the doublet point scheme. The application of the hybrid code to test cases shows an increase in computational efficiency in comparison to a comparable doublet lattice code with no degradation in accuracy.

Nomenclature

a	= speed of sound
ΔA_i	= box area
b	= semichord
F_i	= $L_i \Delta A_i$, box load
k	= reduced frequency, $\omega b/U$
$(1/8\pi)K[M,k,x_0,y_0]$	= kernel function defining nondimensional normalwash at x,y due to point force at ξ,η
L_i	= constant nondimensional lift per unit area in box i
M	= Mach number, U/a
N	= number of boxes
$\Delta p(\xi,\eta)/q$	= nondimensional lifting pressure
q	= dynamic pressure, $\rho U^2/2$
U	= freestream velocity
$w(x,y)$	= normalwash at point x,y
x,y	= point on lifting surface where normalwash is determined
x_0	= $x-\xi$
y_0	= $y-\eta$
Y_{K_c}	= spanwise location of the center of the kth strip, which is part of the summation process
$\delta(\eta-\eta_i),\delta(\xi-\xi_i)$	= Dirac delta function
η_{i_c}	= spanwise location of the center of the ith strip, for which I_i is evaluated
ξ,η	= position on lifting surface where lift is defined
ξ_i,η_i	= position of the load point in the box i (¼ chord, midspan in the box)
ρ	= freestream flow density
$2\sigma_i$	= width of strip i for which I_i is evaluated
$2\sigma_k$	= width of kth strip, numbered sequentially from the root to the tip
ω	= circular frequency, rad/s

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*Curators' Professor. Associate Fellow AIAA.
†Senior Technical Specialist. Associate Fellow AIAA.

Introduction

FOR calculation of subsonic, compressible aerodynamic forces on harmonically oscillating lifting surfaces, the most generally used scheme is the doublet lattice method (DLM).[1-3] The DLM has proven useful for both planar and nonplanar configurations of multiple interacting lifting surfaces of fairly general geometry. In the DLM, the surfaces are represented by a grid of boxes of trapezoidal shape. At the ¼ chord of each box is located a line of pressure doublets with a constant strength, which is the unsteady lift per unit length of the line. The normalwash at the control points for all boxes in the grid is computed using the kernel function representation of Watkins et al.[4] with extension to nonplanar surfaces by Landahl.[5] The control points are at the ¾-chord point, midspan for each box.

Recently, Ueda and Dowell[6] introduced a new approach to the discrete representation of the lifting surface integral equation, which draws heavily on earlier work of Houbolt.[7] This approach, the doublet point method (DPM), also uses a grid of trapezoidal boxes to represent the lifting surface. Instead of doublet lines on box ¼ chords to represent the lift distribution, a single doublet on the box ¼ chord, midspan, represents the lift force on the box. The normalwash at control points is handled in principle as in the DLM. However, because of the well-known singular behavior of the normalwash directly downstream of load points (doublets), an averaging procedure is used, which introduces an improper integral that can be handled by the method described by Mangler.[8]

In the DPM only the singular contribution to normalwash directly behind load points is averaged. Because of this, the computation scheme is very efficient. For planar surface test cases, a code based on the basic structure of the DLM, but with the DPM used for the actual computation of aerodynamic influence coefficients, has been found to run 25–35% faster than a generally available version of the DLM[9,10] with equivalent accuracy.

All of the examples cited in Ref. 6 deal with planar surfaces and use grids which have a uniform strip width. In applications it is often necessary to use unequal strip widths to define control surface edges or other geometric nonuniformities. It was discovered that the DPM does not yield satisfactory results near regions where there is a change in the grid strip width. Figure 1 illustrates the problem which arises in the case of a simple rectangular planform with 15 strips and 300 boxes. The strips are arranged so that the five strips nearest the root are of uniform width of 10% of the span. The remaining 10 strips near the tip are 5% of the span. The effect on the calculation

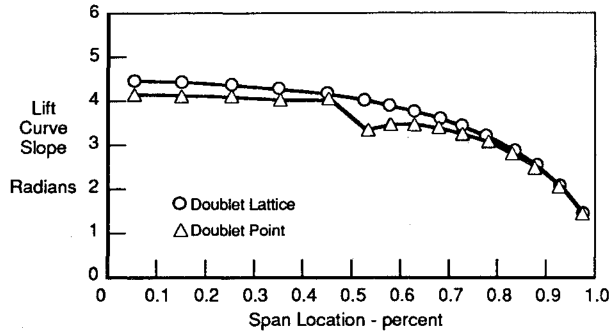

Fig. 1 Comparison of DPM and DLM for rectangular test wing with nonuniform strip distribution, steady-state conditions; $M = 0.0$, $K = 0.0$.

of the spanwise variation of the steady-state lift curve slope ($k = 0.0$) is shown in the figure where the results of DLM calculations (actually vortex lattice) are compared with DPM calculations. The DPM results show a severe discrepancy in the region where the transition in strip width occurs. In an investigation of this problem with the DPM, it was found that a pivotal calculation in the method is valid only if the strip widths are constant and only if the surface is planar. The cases of unequal strips and nonplanar configurations were not considered explicitly by Ueda and Dowell.[6]

In this discussion we review the formal development of the DPM to isolate the difficulty and then describe two methods to eliminate it. The first method is suitable for planar configurations. A second approach, in which elements of both the DLM and DPM methods are used, eliminates all of the problems described and leads to a computational scheme as flexible as the DLM but with most of the efficiency advantage of the DPM.

Review of the Doublet Point Method

The fundamental nature of the difficulty in the DPM can be isolated by considering the planar case. The integral equation relating lift distribution to normalwash is[4]

$$\frac{w(x,y)}{U} = \frac{1}{8\pi}\iint \frac{\Delta p(\xi,\eta)}{q} K[M,k,x_0,y_0]\,\mathrm{d}A$$

The geometry of the planar case is shown in Fig. 2 where a simple grid is shown with a typical box isolated to locate the doublet or load points (sending points) and the normalwash or control points (receiving points).

For the DPM the lift is distributed at load points within each box in the grid:

$$\frac{\Delta p(\xi,\eta)}{q} = \sum_{i=1}^{N} F_i \delta(\xi - \xi_i)\delta(\eta - \eta_i)$$

The box load is

$$F_i = \frac{\Delta P_i}{q}\Delta A_i = L_i \Delta A_i$$

The integral equation is then written

$$\frac{w(x_j,y_j)}{U} = \frac{1}{8\pi}\iint \sum_{i=1}^{N} F_i \delta(\xi - \xi_i)\delta(\eta - \eta_i) K(M,k,x_0,y_0)\,\mathrm{d}\xi\,\mathrm{d}\eta$$
$$= 8\pi \sum_{i=1}^{N} \frac{\Delta p_i}{q} \Delta A_i K\left[M,k,(x_j - \xi_i),(y_j - \eta_i)\right] \qquad (1)$$

The x_j, y_j is the jth control point located at the ¾ chord, midspan of a receiving box.

Equation (1) defines normalwash in an apparently reasonable way except when $y_j = \eta_i$ and $x_j > \xi_i$, that is, when the control point is directly behind the load point. The singularity for this case is easily seen when one considers the form of the kernel[6]

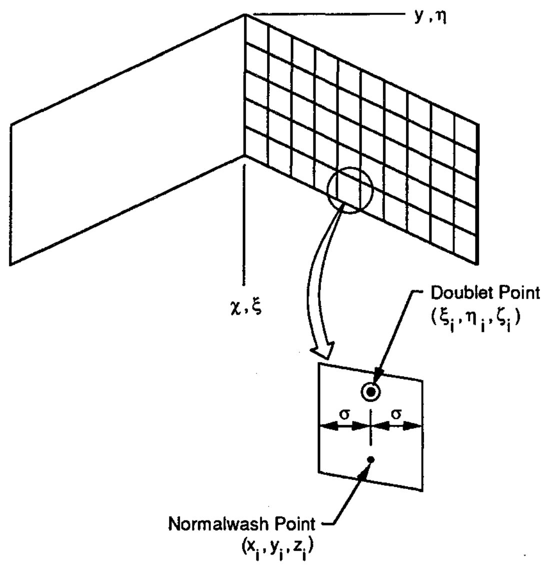

Fig. 2 Geometric relationship for the DPM.

$$K[M,k,x_0,y_0] = e^{-ikx_0}\frac{Me^{ikX}}{R\sqrt{X^2+r^2}} + B \qquad (2a)$$

$$B(k,r,x) = \int_{-\infty}^{X} \frac{e^{ikv}}{(r^2+v^2)^{3/2}}\,\mathrm{d}v \qquad (2b)$$

$$r^2 = (y-\eta)^2, \qquad X = \frac{x_0 - MR}{\beta^2} \qquad (2c)$$

$$R = \sqrt{x_0^2 + \beta^2 r^2}, \qquad \beta^2 = 1 - M^2 \qquad (2d)$$

The singularity is in the integral $B(k,r,X)$, which with a change of variable $u = v/r$, can be written

$$B(k,r,X) = \frac{1}{r^2}\int_{-\frac{X}{r}}^{\infty} \frac{e^{-ikru}}{(1+u^2)^{3/2}}\,\mathrm{d}u$$

In this form it can be seen that as $r \to 0$, $B(k,r,X)$ has the limiting form

$$B(k,r,X) = \frac{1}{r^2}\int_{-\infty}^{\infty} \frac{1}{(1+u^2)^{3/2}}\,\mathrm{d}u = \frac{2}{r^2} \qquad (3)$$

This singular behavior occurs only if $X > 0$, as can be deduced from Eq. (2). For $r = 0$ this can occur only if $x_0 > 0$, that is, for $x_j > \xi_i$.

In the DPM the singular normalwash is smoothed out by averaging across the box width 2σ for control points downstream of load points. That is, one defines the average

$$\langle B(k,r,X)\rangle = \frac{1}{2\sigma}⨎_{-\sigma}^{\sigma}\left[\int_{-\infty}^{X} \frac{e^{ikv}\,\mathrm{d}v}{(r^2+v^2)^{3/2}}\right]\mathrm{d}y_0 \qquad (4)$$

where ⨎ signifies the treatment of the improper integral discussed by Mangler.[8] Since Eq. (3) defines the singular behavior of $B(k,r,X)$, Eq. (4) is conveniently written

$$\langle B(k,r,X)\rangle = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\left[B(k,r,X) - \frac{2}{r^2}\right]\mathrm{d}y_0 + \frac{1}{2\sigma}⨎_{-\sigma}^{\sigma}\frac{2}{r^2}\,\mathrm{d}y_0 \qquad (5)$$

In Eq. (5) the first integral is regular, although, as discussed by Ueda and Dowell,[6] a considerable amount of analysis is required to place it in a form for evaluation.

The improper integral is treated in the sense of Mangler,[8]

$$\int\!\!\!\!\!\!\times_{-\sigma}^{\sigma} \frac{2}{r^2}\,\mathrm{d}y_0 = \int\!\!\!\!\!\!\times_{-\infty}^{\infty} \frac{2}{r^2}\,\mathrm{d}y_0 - \int_{-\infty}^{-\sigma} \frac{2}{r^2}\,\mathrm{d}y_0 - \int_{\sigma}^{\infty} \frac{2}{r^2}\,\mathrm{d}y_0$$

$$= \lim \left\{ \int_{-\infty}^{-\epsilon} \frac{2}{r^2}\,\mathrm{d}y_0 + \int_{\epsilon}^{\infty} \frac{2}{r^2}\,\mathrm{d}y_0 - \frac{4}{\epsilon} \right\} - 4\int_{\sigma}^{\infty} \frac{\mathrm{d}r}{r^2} \qquad (6)$$

Hence,

$$I = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma} \frac{2}{r^2}\,\mathrm{d}y = \frac{2}{\sigma}\int_{\sigma}^{\infty} \frac{\mathrm{d}r}{r^2} \qquad (7)$$

The analytical result in Eq. (7) is

$$I = \frac{1}{2\sigma}\int\!\!\!\!\!\!\times_{-\sigma}^{\sigma} \frac{2}{r^2}\,\mathrm{d}y_0 = -\frac{2}{\sigma^2}$$

When Ueda and Dowell[6] use this result in Eq. (5), they find that the resulting computations for aerodynamic influence coefficients do not compare favorably with equivalent DLM results. The argument is then presented that the integral should not be represented by its analytical result but rather by the discretized result:

$$I = -\frac{2}{\sigma}\sum_{n=1}^{\infty} \frac{1}{r_i^2}\,\Delta r_i = -\frac{2}{\sigma}\sum_{n=1}^{\infty} \frac{2\sigma}{(2n\sigma)^2} = -\frac{\pi^2}{6\sigma^2} \qquad (8)$$

This can be seen to be obtained by taking the integrand to be constant and equal to the value at the center of intervals of width 2σ. It can be argued that this evaluation is consistent with the definition of load points and control points at the midspan of boxes of width 2σ and that the integral is approximated on the actual box grid.

When Eq. (8) is used in the DPM, the computation of aerodynamic influence coefficients for planar surfaces compares favorably with DLM results as shown by Ueda and Dowell.[6] However, Eq. (8) is based on the assumption that the box grid has strips of equal width, and these are the only cases tested by Ueda and Dowell.[6] As seen in Fig. 1, in the case of unequal strip widths, the results of the DPM do not compare with the DLM near the transition in strip width. This problem can be traced directly to the discrete result of Eq. (8), which depends on using equal strip widths. It has been determined that for planar surfaces the integral of Eq. (7) must be evaluated in discrete form based on the actual distribution of strip widths.

Modification for Nonuniform Strips

In the planar case, the modification to account for unequal strips is straightforward and consists only of a discrete representation of Eq. (7) based on evaluation of the integrand at

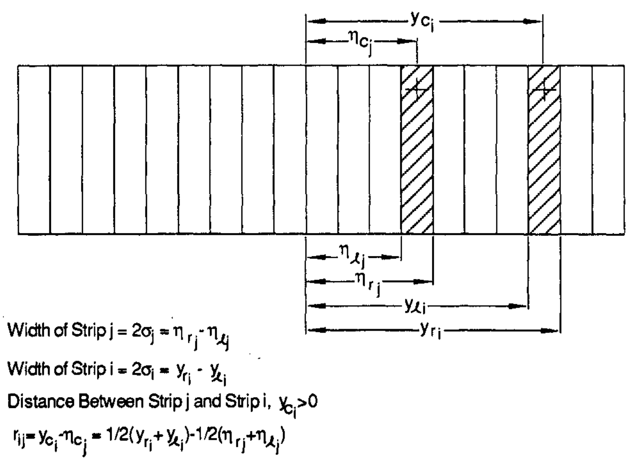

Fig. 3 Geometric definitions for the modification to the DPM to account for nonuniform strip distribution.

box centers and taking into account the unequal strip widths. In addition, a choice must be made on a grid extension beyond the tip of the surface. The results for aerodynamic influence coefficients are only slightly sensitive to this choice, and so the convenient strategy is to assume the grid continues beyond the tip with strips of width equal to that of the tip strip.

To form the discrete representation of Eq. (7), it must be written in the original form

$$I = -\frac{1}{2\sigma}\left[\int_{-\infty}^{-\sigma} \frac{2}{r^2}\,\mathrm{d}y_0 + \int_{\sigma}^{\infty} \frac{2}{r^2}\,\mathrm{d}y_0\right] \qquad (9)$$

to allow for asymmetry of box widths in the discrete representation. The equivalent to Eq. (8) when the strips are unequal is

$$I_i = -(1/2\sigma_i)\left[2(S_1 + S_2)\right] \qquad (10)$$

where

$$S_1 = \sum_{k=i+1}^{\infty} \frac{2\sigma_k}{(y_{k_c} - \eta_{i_c})^2} \qquad (11)$$

$$S_2 = \sum_{k=1}^{i-1} \frac{2\sigma_k}{(y_{k_c} - \eta_{i_c})^2} + \sum_{k=1}^{\infty} \frac{2\sigma_k}{(y_{k_c} + \eta_{i_c})^2} \qquad (12)$$

Figure 3 shows the pertinent geometry for arriving at Eqs. (10–12). S_1 is the evaluation of the integral from σ to ∞, and S_2 is the evaluation of the integral from $-\infty$ to $-\sigma$. The summations converge fairly rapidly, and a set number of terms, for example, 100, is taken. The result for aerodynamic influence coefficients is not sensitive to small changes in S_1 and S_2 due to the choice of the number of terms. In fact, it becomes obvious that only the effect of boxes close to box i is important.

The integral I_i is only dependent on the strip layout in the box grid and is a property of strip i. The integral I_i is calculated at the geometry processing phase of the computer code. The integral adds almost nothing to the computation time so that this modification does not alter the observed efficiency of the DPM. As stated earlier, the code we have developed uses the basic structure of a standard DLM code. The modification described had to be inserted in this code with proper care given to keeping track of the strip numbering scheme, particularly for tandem coplanar surfaces and cases of multiple panels describing a single surface.

The necessity for this modification in the case of nonuniform strips and the observation of the critical dependence of the accuracy of the DPM on the consistent discrete representation of the integral of Eq. (7) gives rise to serious doubt about a simple extension to noncoplanar and intersecting surfaces. A method of interpreting the consistent discrete representation of Eq. (7) for, say, a simple lifting surface with an inboard panel in the x,y plane and with an outboard panel which has a dihedral angle is not clear. The interpretation becomes even more uncertain when one considers intersecting surfaces such as a T-tail. The conclusion is that the modification described in this section cannot be easily implemented for other than planar configurations.

Results of the Modification

Verification of the modification has been carried out on a test planform of rectangular shape of aspect ratio 2 with a baseline grid of 10 equal strips. Variations are derived by using 12 strips, in one case with four 5% span strips at midspan and in the other case with four 5% span strips at the tip. Figure 4 shows the results of several computations of spanwise variation of lift for steady state. The baseline of 10 evenly spaced strips is shown with the solid curve. This is in good agreement with DLM calculations. The discrepancy created with unequal strips for the original DPM in the center of the span is clearly shown by the triangular symbols. The improvement for this case when the modification is used is shown by the square

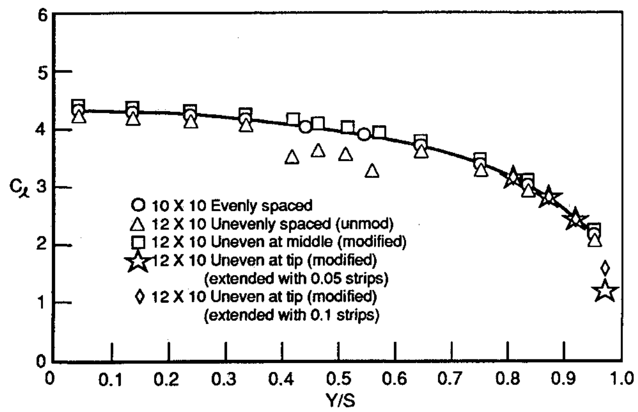

Fig. 4 Incompressible steady-state lift curve slope for several versions of the modified DPM applied to the rectangular test wing; $M = 0.0$, $K = 0.0$.

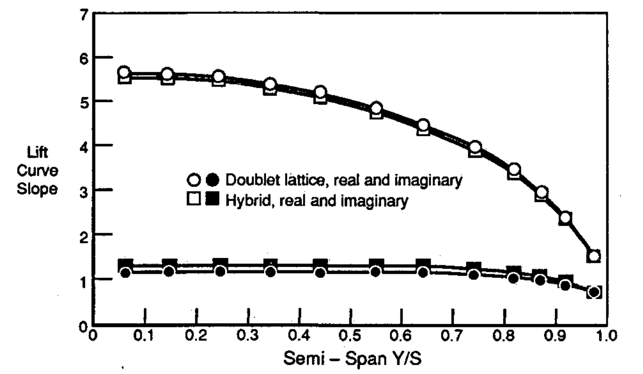

Fig. 5 Comparison of DLM and DHM for rectangular wing oscillating in pitch about the ¼ chord; real and imaginary part of the spanwise distribution of lift curve slope; uneven spacing at wing tip, 12×10; $M = 0.8$, $K = 0.3$.

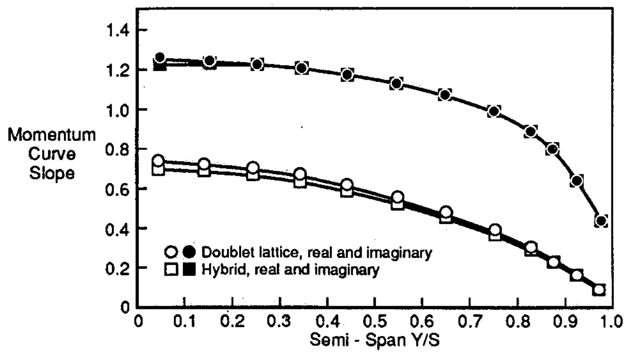

Fig. 6 Comparison of DLM for rectangular wing oscillating in pitch about the ¼ chord; real and imaginary part of the spanwise distribution of aerodynamic moment curve slope; uneven spacing at wing tip, 12×10; $M = 0.8$, $K = 0.3$.

symbols. A second test is made when the uneven strips are positioned at the tip. As shown in Fig. 4, when the modification is used the lift distribution is about the same as for the basic planform. Note that for this case we have extended the summations beyond the tip using both the large strips and the small strips, and the results are only slightly sensitive to this choice. The correction is not dependent on Mach number or reduced frequency, and so no comparisons for other cases are shown here. A number of test cases at other M and k conditions have proven the modification to be equally effective.

Role of Averaging in the Doublet Point Method

As noted in the previous discussion, the simple modification based on a consistent discrete representation of Eq. (7) appears to depend principally on accounting for the box widths of strips located near the strip i for which I_i is to be calculated [Eqs. (9-12)]. This has given rise to the suspicion that it is not really the analytical evaluation of Eq. (7) that is inappropriate in the DPM [thus creating the necessity to introduce the discrete form, Eq. (8)]. It seems highly likely that the real problem lies in not properly handling receiving boxes which are very close to sending boxes and for which $r^2 = (Y_i - \eta_j)^2$ is therefore a small but nonvanishing quantity.

The basic principle in the DPM is that the kernel function relationship can be used directly to relate normalwash at control point x_j, y_j to a point doublet at ξ_i, η_i except for the case of receiving boxes directly downstream of sending boxes. In this case a partial averaging procedure is used in which only $B(k,r,X)$ is averaged, and even then only the singular part and a few additional easily averaged nonsingular contributions.[6]

Houbolt[7] proposed what essentially amounts to the DPM with the idea of averaging over all the boxes. In the DPM the implicit assumption is therefore that averaging is necessary only for the case of receiving boxes behind sending boxes. All other receiving/sending configurations can be treated under the supposition that direct evaluation of the kernel for the load point and control point coordinates is near enough to computing the normalwash averaged spanwise on the receiving box. Principally due to the singular behavior of $B(k,r,X)$, which is of the form $1/r^2$, treating the point evaluation of $B(k,r,X)$ as the average for receiving boxes near sending boxes is not accurate enough. It is this inaccuracy, and not the analytical evaluation of Eq. (7), which forces the discrete representation of Eq. (7). It seems that the discrete evaluation of Eq. (7) offsets the basic shortcoming.

The normalwash averaging concept can be extended to sending and receiving box pairs which are within one or two strip widths of one another. This has not been carried out in analytic form but has been done numerically using Gaussian quadrature. For best results the average is taken spanwise on the receiving box over a length equal to the sending box width. When this is done and the averaging procedure of the DPM is used for the case of box pairs in the same strip, the difficulties associated with the unequal strip widths disappear without the necessity of using a discrete evaluation of the singular integral in Eq. (8). It is easy to visualize that this averaging procedure is similar to the calculation in the DLM, which evaluates the normalwash of the control point of box j due to the doublet line on box i.

Nonplanar Case

When sending and receiving boxes are not coplanar the kernel becomes[5]

$$K[M,k,x_0,y_0] = e^{-ikx_0}\left\{\frac{Me^{ikX}}{R\sqrt{X^2+r^2}} + B\right\}T_1$$

$$- r^2 e^{-ikx_0}\left\{\frac{ikM^2e^{ikX}}{R^2\sqrt{X^2+r^2}} + 3I + \frac{Me^{ikX}}{R^2(X^2+r^2)^{3/2}}\left[\frac{\beta^2(X^2+r^2)}{R} + 2R - MX\right]\right\}T_2 \qquad (13)$$

where $B(k,r,X)$ was previously defined in Eq. (2) and

$$I(k,r,X) = \int_{-\infty}^{X} \frac{e^{ikv}}{(r^2+v^2)^{5/2}}\,\mathrm{d}v$$

$$T_1 = \cos(\gamma_r - \gamma_s)$$

$$T_2 = \frac{1}{r^2}(Z_0\cos\gamma_r - Y_0\sin\gamma_r)(Z_0\cos\gamma_s - Y_0\sin\gamma_s) \qquad (14)$$

The Y_0 and Z_0 are the y and z distance between sending and receiving points $Y_0 = Y_r - Y_s$, $Z_0 = Z_r - Z_s$, and γ_r and γ_s are the dihedral angles of the surfaces at the receiving and sending points. T_2 will be nonzero if sending and receiving points are not coplanar. This second part of the kernel with T_2 as a factor is commonly called the nonplanar kernel.

It would normally be thought that for noncoplanar box pairs it would be unnecessary to take special care in application of the DPM. However, it is important to note that the integral, Eq. (14), can be written

$$I(k,r,X) = \frac{1}{r^4}\int_{-\frac{X}{r}}^{\infty} \frac{e^{-ikru}}{(1+u^2)^{5/2}}\,du \tag{15}$$

showing a behavior proportional to $1/r^4$ for small r. Now this never contributes in a truly singular way because, for receiving boxes directly behind sending boxes, $T_2 = 0$, and the nonplanar portion of the kernel vanishes. However, for receiving boxes near sending boxes, the contribution of $I(k,r,X)$ can vary rapidly with r, which makes a direct evaluation of $I(k,r,X)$ an inaccurate approximation to the average value over the box. This situation would occur near the intersection of noncoplanar panels.

Reference 6 did not specifically address this problem nor discuss how to interpret the discrete evaluation of the integral of Eq. (6) when the lifting surface has panels of different dihedral or the more extreme case of perpendicular surfaces such as in the case of a T-tail. In a thesis,[11] Weir set up the DPM for noncoplanar surfaces. In this implementation the special treatment of Eq. (6) was handled just as if the surface containing the sending box were completely planar to infinity. That is, the integral of Eq. (7) was evaluated in the discrete form of Eq. (8). Example calculations based on a simple T-tail did not produce good comparisons with calculations based on the DLM. In the geometry chosen this could have been due to a difference in strip width for the horizontal and vertical tail and also due to the behavior of the integral of Eq. (15) near the intersection of the two surfaces.

The extension of the DPM to noncoplanar and intersecting surfaces is seen to be complicated by factors not originally addressed in Ref. 6. We have shown here how one of the problems can be resolved for the coplanar case. There does not appear to be a practical way of applying the same kind of thinking to the noncoplanar case. An entirely different approach is required.

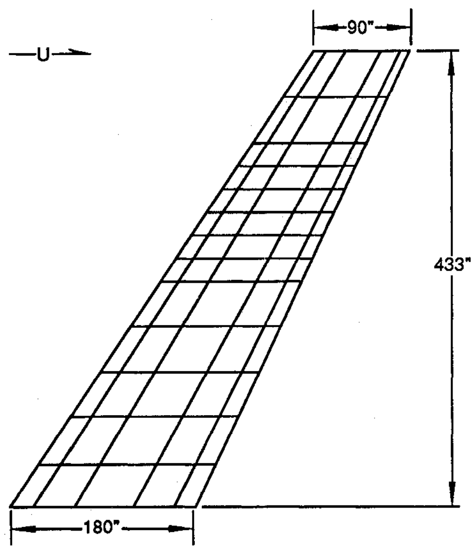

Fig. 7 Geometry of the high aspect ratio 30-deg swept wing planform.

Hybrid Formulation

It was previously noted that, by extending the averaging idea to sending/receiving box pairs that are close according to some measure, it is possible to use most of the ideas of the DPM in a formulation which is insensitive to variations in strip width. This observation lends support to the conjecture that the difficulty in the DPM is an inconsistent treatment of sending/receiving box pairs which are close enough that the $1/r^2$ singularity in $B(k,r,X)$ is important. This leads one to conclude that in the nonplanar kernel the $1/r^4$ singularity in $I(k,r,X)$ [which is multiplied by r^2, see Eq. (13)] will introduce the same problem when $T_2 \neq 0$.

Because of the success of the averaging and because of the similarities of the averaging calculations to calculations in the DLM, it becomes apparent that the best approach to the DPM is to use pointwise kernel evaluations for sending/receiving box pairs which are distant and to use the DLM calculations for sending/receiving box pairs which are close. There are a number of measures of closeness which could be used. In the implementation discussed here, close is defined as less than two strip widths, the strip width being the larger of the sending and receiving strip widths.

Validation of the Hybrid Formulation

The creators of the DLM and the DPM have previously made extensive validations of their computational schemes by comparison with available experimental measurements.[1-3,6] In the present study, validation of the hybrid method, described as the doublet hybrid method (DHM), is accomplished by comparison with the DLM, which is widely accepted. The first comparison considers the rectangular planform of aspect ratio 2 with eight 10% span strips and with the tip region divided into four 5% span strips. There are 10 boxes per strip. It was previously shown in Figs. 1 and 4 that this arrangement is not modeled satisfactorily by the unmodified DPM, but that a simple modification which reinterprets a key calculation in the DPM will lead to good comparison between the DLM and DPM for lift distribution. We have made comparisons of the

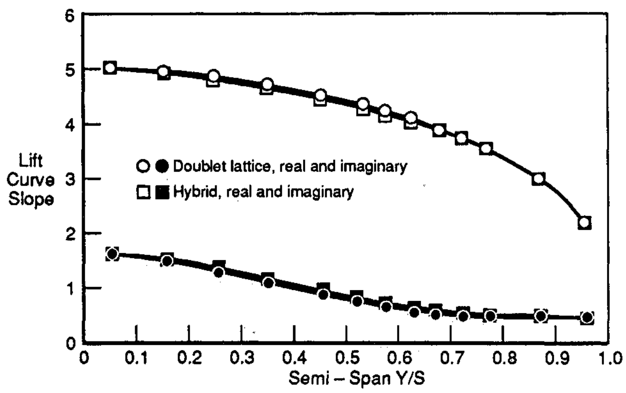

Fig. 8 Comparison of DLM and DHM for high aspect ratio swept wing in rigid twist about the elastic axis at ½ chord; real and imaginary part of the spanwise distribution of lift curve slope; uneven spacing in middle, 13×5; $M = 0.8$ and $K = 0.5$.

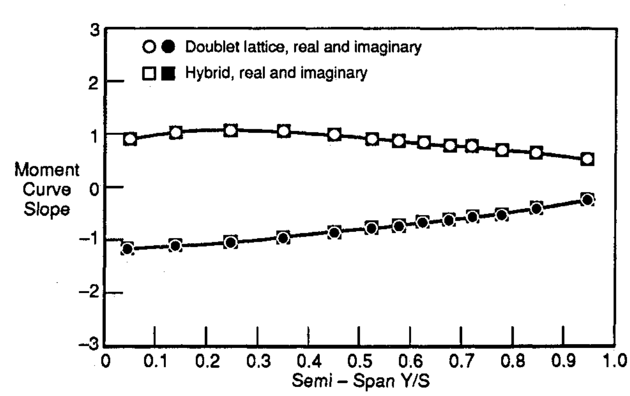

Fig. 9 Comparison of DLM and DHM for high aspect ratio swept wing in rigid twist about the elastic axis at ½ chord; real and imaginary part of the spanwise distribution of aerodynamic moment coefficient; uneven spacing in middle, 13×5; $M = 0.8$ and $K = 0.5$.

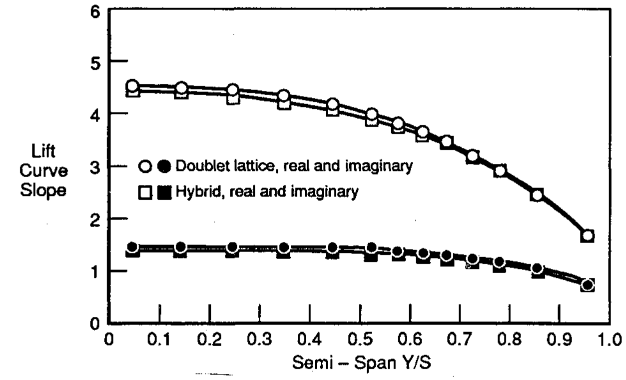

Fig. 10 Comparison of DLM and DHM for low aspect ratio swept wing in rigid twist about the elastic axis at ½ chord; real and imaginary part of the spanwise distribution of lift curve slope; uneven spacing in middle, 13×5; $M = 0.8$ and $K = 0.5$.

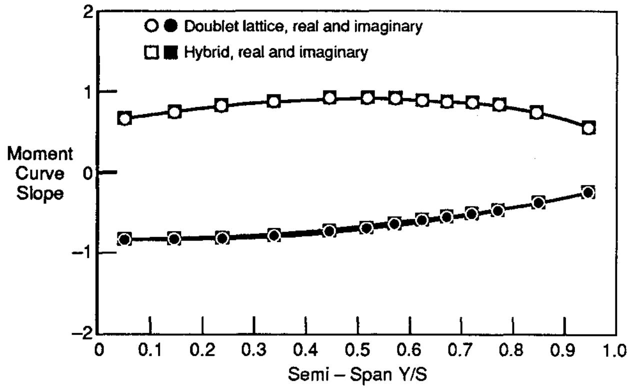

Fig. 11 Comparison of DLM and DHM for low aspect ratio swept wing in rigid twist about the elastic axis at ½ chord; real and imaginary part of the spanwise distribution of aerodynamic moment coefficient; uneven spacing in middle, 13×5; $M = 0.8$ and $K = 0.5$.

DLM and the new DHM in terms of the spanwise lift distribution and the spanwise moment distribution for harmonic motion in pitch about the ¼ chord at a condition of $M = 0.8$ and $k = 0.3$. These are shown in Figs. 5 and 6, where it is noted that good agreement is obtained.

A second planform is shown in Fig. 7. This is a relatively high aspect ratio swept wing with a nonuniform patch of strips in the interior portion of the wing. This is a very coarse chordwise grid for the combination of Mach number $M = 0.8$ and reduced frequency $k = 0.5$ chosen for the comparison. Figures 8 and 9 show that the spanwise distribution of lift and moment for harmonic motion in rigid rotation about the elastic axis at ½ chord as determined by the DLM are almost duplicated by the DHM.

A third example is a low aspect ratio swept wing derived from the wing of Fig. 7 by clipping off the tip at the half-span. The resulting span is 216.5 in., and the tip chord is 135.5 in. This new planform is initially divided into 10 10% span strips, with five boxes per strip. Strips 6, 7, and 8 are then subdivided into 5% span strips. The conditions chosen for calculation of the lift and moment distribution due to harmonic oscillations in rigid rotation about the elastic axis at ½ chord are $M = 0.8$ and $k = 0.5$. Figures 10 and 11 show a good comparison between the results obtained from the DLM and those from the DHM.

All of the results shown in Figs. 5–11 could have been nearly duplicated by using the modified DPM method, with the modifications defined by Eqs. (10–12). As noted previously, this modification is not believed to be applicable to configurations which are not coplanar. The hybrid approach is, however, directly applicable to nonplanar configurations and is in fact applicable to the complete range of configurations for which the DLM is used.

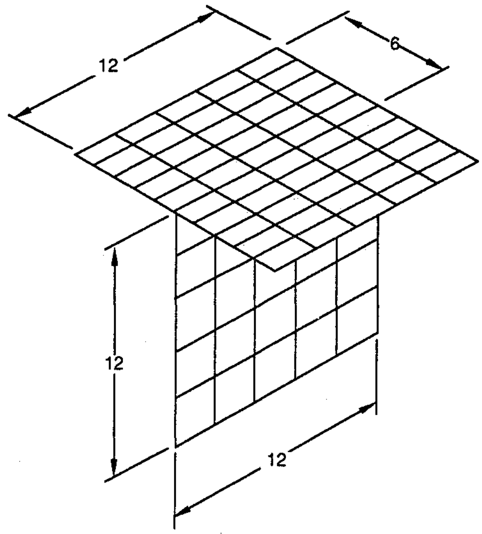

Fig. 12 Geometry of T-tail test configuration.

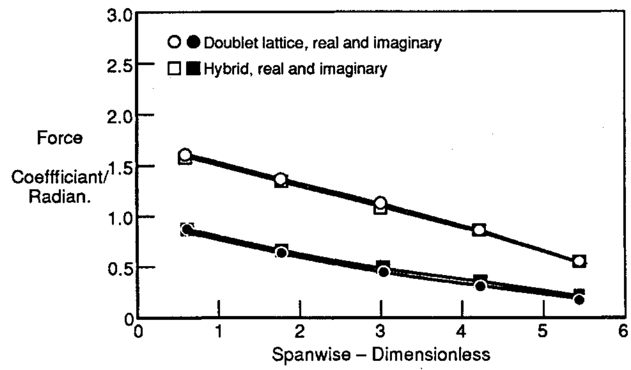

Fig. 13 Comparison of DLM and DHM for T-tail configuration oscillating in yaw about the midchord of the vertical fin; spanwise distribution of lift coefficient on the horizontal tail per unit angle of yaw of the vertical fin; $M = 0.3$ and $K = 0.5$.

To validate the DHM for such configurations, the T-tail configuration of Fig. 12 has been considered. This configuration is recognized as one for which experimental data for yaw and sideslip derivatives have been obtained by Clevenson and Leadbetter.[12] Giesing et al. used these data in validating their DLM formulation.[10] These comparisons can only be described as marginal, but so few data are available that it is difficult to quantify the success with which the doublet distribution methods can predict the air loads on interfering surfaces. Here we address only the comparison of the DLM and DHM at $M = 0.3$ and $k = 0.5$ for oscillations in yaw of the configuration about the vertical midchord axis of the fin. Figure 13 shows the spanwise lift distribution on the horizontal tail per unit angle of attack of the vertical fin. The comparison is noted to be very close, and this is true for other measures of load distribution.

In all cases tested, the comparison between the DLM and the DHM is very good, and one can conclude that both methods offer essentially equivalent accuracy in representing solutions of the governing integral equation which relates loading to normalwash. The DHM retains most of the efficiency of the DPM. As noted previously for configurations similar to those treated here, the DPM was found to run about 30% faster than the DLM. This is because of the greatly reduced number of kernel evaluations. This is, of course, compromised to some extent in the DHM because of the extra kernel evaluations associated with close box pairs. However, for the configurations described here, the DHM is still about 25% more efficient than DLM. This observation will be affected by the configuration to some extent. For example, the algorithm currently used invokes the lattice normalwash evaluations for close box pairs that are less than or equal to two box widths apart. Therefore, little advantage is to be expected from DHM if only a few strips are used to model the surface.

Conclusion

In this investigation a problem in the DPM for subsonic lifting surfaces has been identified. The difficulty arises for planar lifting surfaces when strips of unequal width are required for modeling. This is due to an inconsistent use of the basic normalwash averaging premise on which the method is based. It is noted that a consistent use of the averaging principle for the entire planform will substantially reduce problems with unequal strip widths. A simple modification of DPM has been devised that will allow treatment of unequal strips in the planar case, but the direct extension to nonplanar and interfering surfaces does not appear promising. This has suggested the use of a hybrid scheme in which the best features of the DLM and the DPM are used. It has been demonstrated that a meaningful increase in the efficiency of computations can be achieved at very little cost in accuracy with the implementation of this hybrid approach. The approach is applicable to the same range of subsonic configurations as is the original DLM.

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